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by

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On Analysis of Viscoelastic Structure

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Submitted in partial fulfillment of the
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ABSTRACT

This study is to understand the effects of nonlinear behavior of structures using a finite element method.

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In particular, large displacement problems and visco-elastic problems remain a challenging engineering problems today. The viscoelastic problems depend on the relaxation function which is the source of material nonlinearities.

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I. INTRODUCTION

A. ANALYSIS OF STRUCTURES

Analysis of structures subjected to large displacements requires the inclusion of nonlinear terms in the strain displacement relations. In the formulation of the problem, either total Lagrangian or Eulerian approach is adopted. In order to assure structural integrity for loads beyond linear range, nonlinear analysis is very much mandated.

Another class of problems is the viscoelastic behavior of structures - problems where stresses in the components vary with time at a constant deformation state or strains vary with time at constant stress states.

These problems as applied to simple one-dimensional structures will be addressed in this research work. In what follows is a brief survey of literature in this area. At the end of this chapter, overview of the thesis is described.

B. LITERATURE REVIEW

A method for nonlinear static and dynamic analysis using cable and beam elements is given by Bergan et al [1]. Formulation allows for material nonlinearities and very large deformations. Gadala and Oravas [2] present a comparison of finite element formulations based on the

Lagrangian, updated Lagrangian and Eulerian formulations for nonlinear mechanics problems.

Henriksen [3] applies a single integral constitutive law to describe viscoelastic behavior in his formulation. Ghoneim et al [4] treat the total strain rate as sum of viscoelastic and viscoplastic contributions in implementing the constitutive equations in their formulation, Pinsky and Kim [5] use variational approach for studying the nonlinear viscoelastic shell behavior by representing the layered shell as a multi-director field. The formulation includes, within individual layers, the effects of transverse shear and transverse normal strain to arbitrary orders in the layer thickness coordinate [6, 7].

Saigel and Yang present a quadrilateral shell element and describe implementation of elastic-viscoplatic constitutive law for large deformation. A large collection of papers on the application of ADINA may be found in [8].

Response of nonlinear mechanical systems to random excitation is given by To [9]. Thermomechanical response of uniaxial bars with thermoviscoplastic constitutive laws is discussed by Allen [10]. Large deformations of the contacting bodies on the contact surfaces and solution methods are given consideration in [11, 12]. Large deformations of composite beams subjected to axial dynamic load is reported in [13].

Based on Eulerian-Lagrangian formulation, frictional

contact analysis is discussed by Haber et al [14].

Halleux and Casadei [6] present transient large strain analysis of structures based on biquadratic elements. Polar-decomposition is used in arriving at explicitly derived tangent stiffness matrices for a frame element by Kondoh and Atluri [15]. A method based on co-rotational coordinate system, where the local cartesian coordinate system attached to each element translate and rotate is described by Mattiasson [16].

Inelastic problems based on incremental virtual work equations is given by Nagtegaal [17]. A constant strain rate is assumed during an increment in integrating the constitutive equations.

Planar curved beams undergoing large rotations is analyzed using Helinger-Reissner mixed variational principle by Noor [18]. Steele discusses several nonlinearities including buckling, creep, and nonlinear elastic problems in [19]. The use of Lagrangian formulation using an elasto-plastic model is presented by Voyiadjis and Navaee [20]. Hirashima et al [21] present an approach for the snap-through analysis of curved shell panels and determination of initial stress matrix.

Importance of viscoelastic relaxation effects on a pellet are investigated by Basombrio [22], where it is observed that the duration of the ramp input influences the response considerably. Viscoplastic constitutive models are

obtained explicitly, without resorting to inversion, to be directly implemented in a finite element formulation [23]. An intrinsic time scale that can vary with time and position is used, together with means to control the numerical integration in the approximation of constitutive model by Chambers and Becker [24].

Chandra et al [25, 26] discuss applications of visco-plastic models in the metal-forming. Elasto-plastic models are used in analyzing unilateral contact and friction problems by Cheng et al [12]. Chung et al [27] suggest methods for varying parameters in guiding solutions for the simulation of plastic forming processes. Liu et al [28] propose formulation for path-dependent materials based on arbitrary Lagrangian Euler method and include an elastic-plastic wave propagation problem.
Rheinboldt and Riks describe several algorithms based on continuation method and mesh refinements for the solution of nonlinear finite element equations. Ryu and Arora [29] propose an approach to solve nonlinear finite element equations based on substructures concept.

Several papers pertaining to visco-elastic, visco-plastic behavior and their implementation is reported in [10, 7, 4, 30, 31].

C. THESIS OVERVIEW

The objective of this thesis is to review some of the literature available in the analysis of large deformation of structures, to develop the theory to include large displacements and viscoelastic behavior of one dimensional structures, and in the process, to understand the physics of the effects of nonlinearities. Chapter II presents theoretical formulation based on principle of virtual work and incremental load method. Descretization based on a simplex element is given.

Chapter III gives the algorithms used in the solution procedure. Full Newton-Raphson and Modified Newton-Raphson methods are described.

Chapter IV describes some test cases and numerical results obtained.

Finally, Chapter V provides conclusions and some remarks for future work in this area.

II. THEORETICAL FORMULATION

A. OVERVIEW

In this chapter we consider a nonlinear continuum equations for a rod element. This approach is based on the principle of virtual work [48]. The resulting equations are in convenient form for numerical solution via finite element methods [49]. This, of course, is the conceptual basis of the discretized-solid finite elements, introduced by J.T.Oden et al [50] in the elastic structures and nonlinear continua.

B. NONLINEAR CONTINUUM EQUATIONS

Discuss a rod element with the total Lagrangian Method. It may be proved that internal virtual work is equal to external virtual work. Namely a deformable system is in equilibrium if and only if the total external virtual work is equal to the total internal virtual work for every virtual displacement consistent with the constraints.

1. Principle of Virtual Work

In this section we prove that total internal virtual work is equal to total external virtual work. Consider the concept of minimum potential energy [51]. The condition is satisfied to equilibrium state and the change of potential with respect to the displacement

must remain stationary. And consider the load-displacement and stress-strain relationship of figures shown below.

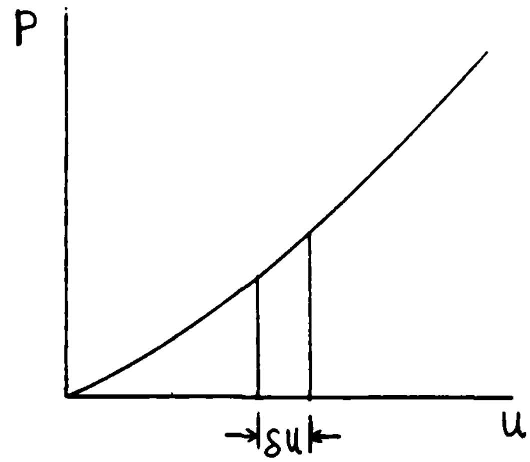


Figure 2.1 Load-Displacement Curve

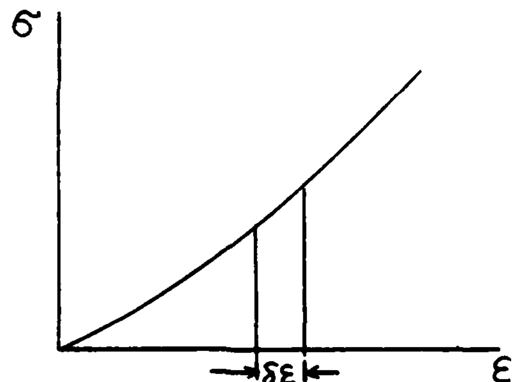


Figure 2.2 Stress-Strain Curve

In General, the total potential energy is equal to the summation of strain energy and potential energy.

ie, $\pi_p = U + W$ (2.1)

where, π_p : Total potential energy
U : Strain energy (internal work)
W : Potential energy of load system (external work)

From **Figure** (2.1),

$$W = - \int p \, \delta u$$

$$= - \Sigma P_i \; u_i \qquad (2.2)$$

where, P_i : i^{th} load
u_i : deflection under i^{th} load

From Figure (2.2),

$$U = \int_{vol} \int_{o}^{\epsilon} \sigma \, \delta\epsilon \; d(vol)$$

$$= .5 \int_{vol} \sigma \; \epsilon \; d(vol) \qquad (2.3)$$

Substituting the equation (2.2) and (2.3) in equation (2.1), we get

$$\pi_p = \int_{vol} .5 \; \sigma \; \epsilon \; d(vol) - \Sigma \; p_i \; u_i \qquad (2.4)$$

Equation (2.4) can be represented by matrix form, the following is the matrix formula.

$$\pi_p = \int_{vol} .5 \; \{\sigma\} \; \{\epsilon\} \; d(vol) - \{ u_i \}^T \{ p_i \} \qquad (2.5)$$

where, $\{\sigma\}$: a vector of stresses
$\{\epsilon\}$: a corresponding vector of strains
$\{u_i\}$: a vector of elemental nodal displacements
$\{p_i\}$: a vector of elemental nodel forces

Let us define [B], a matrix relating strains and displacements,

ie, $\{\epsilon\} = [B] \; \{u\}$ (2.6)

and define [D], a matrix relating stresses and strains,

ie, $\{\sigma\} = [D] \; \{\epsilon\}$ (2.7)

Substituting equation (2.6) & (2.7) in equation (2.5), we get

$$\pi_p = \int_{vol} \{u_i\}^T \{B\}^T [D] [B] \, d(vol) \{u_i\} - \{u_i\}^T \{p_i\} \quad (2.8)$$

Now we consider the minimum potential energy, the condition is as follows:

$$\frac{d\pi_p}{d\{u_i\}} = 0 \quad (2.9)$$

Taking derivatives with respect to $\{u_i\}$ in equation (2.8) and applying to equation (2.9), we get

$$\{p_i\} = \int_{vol} [B]^T [D] [B] \, d(vol) \{u_i\} \quad (2.10)$$

From relating loads and displacements, we can get the elemental stiffness matrix [k].

ie, $$[K] = \int_{vol} [B]^T [D] [B] \, d(vol) \quad (2.11)$$

Summing all the elements of the entire structure, equation (2.10) becomes

$$\{P\} = [K] \{u\} \quad (2.12)$$

where,
$$\begin{aligned} \{P\} &= \Sigma \{P_i\} \\ [K] &= \Sigma [K] \\ [u] &= \Sigma \{u_i\} \end{aligned}$$

2. <u>One Dimensional Finite Element</u>

Consider a shape function which is linearly function with geometric boundary condition with one dimensional

finite element. We define that the shape function, Ψ_N, is

$$\Psi_N(x) = a_N + b_N X$$

In order to derive the constants in the shape function for a rod element with linear displacement field, use the geometric boundary condition.

We may obtain the constants as follows:

$$a_1 = \frac{x2}{Lo}$$

$$b_1 = -\frac{1}{Lo} \qquad (2.13)$$

$$a_2 = -\frac{x1}{Lo}$$

$$b_2 = -\frac{1}{Lo}$$

The relationship between displacement and shape function is given below

$$u = \sum_{N=1}^{M} \Psi_N u_N \qquad (N=1;\ M=2,\ \ldots.)$$

$$= \Psi_1(x)\, u_1(t) + \Psi_2(x)\, u_2(t) \qquad (2.14)$$

Another consideration is the viscoelastic materials. For viscoelastic materials, the stresses may be expressed in terms of relaxation functions as given by

$$S(t) = \int_{-\infty}^{t} G(t - \xi)\, S^{e}(\xi)\, d\xi$$

$$= S^{e}(t) - I(t) \qquad (2.15)$$

where, $S^{e}(t) = S^{e}[r(t)]$: elastic response

$$I(t) = \int_{o}^{t} \frac{G(t-\xi)}{\partial \xi} S^{e}(\xi)\, d\xi$$

G(t): relaxation function.

Assume the element is in equilibrium and quasistatic state. The motion equation for a one dimensional finite element is given by J.T.ODEN [ref.6],

$$\int_{V_o} S\Psi_{N,x} \, (1 + \Psi_{M,x} \, u_M) \, dV_o = P_N(u_M, t)$$

where, $V_o = A_o L_o$: undeformed volume

P_N : total generalized force at node N

On substituting for above shape functions and their derivatives, we may obtain

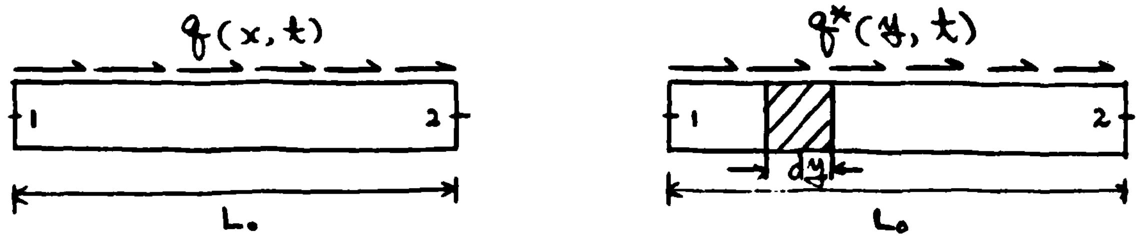
$$f_N(u_M, t, I) = V_o b_N [S^{e} - I](1 + b_k u_k) - P_N(u_M, t) = 0 \qquad (2.16)$$

C. CONSISTENT NODAL FORCES

Consider a rod element in two configurations: undeformed and deformed.

Let the distributed forces per unit length in these configurations be q and q* respectively. The consistent load vector depends on both the magnitude of the load and the deformed state.

One dimensional rod element under distributed load is showed as following figures.



where, Lo : undeformed rod element length

L : deformed rod element length

q(x,t) is distributed load per unit undeformed length

q*(y,t) is distributed load per unit deformed length

Figure 2.3 One-Dimensional Rod Element under Distributed Load.

Consider the relationship between the deformed and undeformed coordinate. It may be written as

$$y(x,t) = x + u(x,t) \tag{2.17}$$

where x: undeformed coordinate of a point

y: deformed coordinate of a point

u: displacement of a point within element

The consistent nodal forces vector is given by

$$\underline{P} = \int_V \rho \hat{F} N \, dV = \int_{V_0} \rho_0 \hat{F}_0 \underline{N} \, dV_0 \tag{2.18}$$

In index notation, nodal forces may be written as

$$P_N = \int_V \rho \hat{F} \Psi_N dV = \int_{V_0} \rho_0 \hat{F}_0 \Psi_N dV_0 \tag{2.19}$$

with $N = 1, 2$

where F : body forces per unit mass deformed

F_o : body forces per unit mass undeformed

ρ : deformed density

ρ_o : undeformed density

Relating $q^*(y,t)$ and $q(x,t)$ is given by

$$q(x,t) = \lambda q^*(y,t) \tag{2.20}$$

where, $\lambda = dy/dx$: stretch

Consider the undeformed body force per unit mass, then we may write

$$\hat{F}_o = \frac{q^* dy}{dm} = \frac{q}{\rho_o A_o} = \frac{\lambda q^*}{\rho_o A_o} \tag{2.21}$$

Using equation (2.17)

$$y = x + \Psi_M u_M \tag{2.22}$$

$$\frac{dy}{dx} = 1 + \Psi_{M,x} u_M \tag{2.23}$$

The undeformed body force per unit mass is then

$$\hat{F}_o = \frac{q^*}{_o A_o} (1 + \Psi_{M,x} u_M) \tag{2.24}$$

The consistent nodal forces take the following form,

$$P_N = \int_o^L q^* \Psi_N dx + \int_o^{Lo} q^* \Psi_{M,x} u_M \Psi_N dx \tag{2.25}$$

Substituting the shape function in equation (2.25), we may get

$$P_N = (1 + b_M u_M) \int_0^{Lo} q^*(a_N + b_N\ x)\ dx \tag{2.26}$$

Now we consider some special cases of equation (2.26).

One case is the uniformly distributed loads.

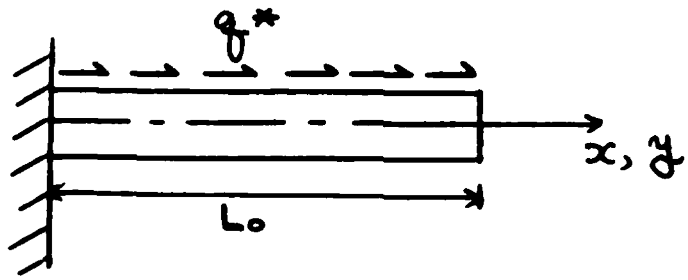


Figure 2.4 Rod Element under Uniformly Distributed Loads

The uniformly distributed loads are given

$$q^*(y,t) = q_o\ \ h(t)$$

Substituting the loads in equation (2.26), we may get the consistent nodal forces.

$$\{\ P_M\ \} = \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix} = \frac{q_o\ h(t)}{2}\,[Lo + u_2 - u_1] \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} \tag{2.27}$$

where, $\quad h(t) = \dfrac{du}{dx}$

Another case is the linearly distributed loads.

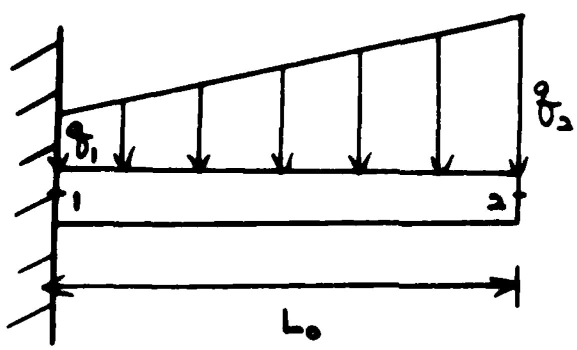


Figure 2.5 Rod Element under Linearly Distributed Loads

Linearly distributed loads are given

$$q(x,t) = q\ (t) + [q\ (t) - q\ (t)]\left(\frac{y}{L_o}\right)$$

Substituting the loads in equation (2.26) we may get the consistent nodal forces.

$$\{\ P_N\ \} = \begin{Bmatrix} p_1 \\ p_2 \end{Bmatrix} = \frac{(L_o - u_1 + u_2)}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} \quad (2.28)$$

D. SOLUTION METHODS

In this section we derive the equilibrium equations of incremental method, Full Newton-Raphson, and Modified Newton-Raphson Method.

It is based on total Lagrangian Method and use the Taylor Serier at a given equilibrium point.

The element stiffness matrices and consistent load vectors are assembled to form global matrices in the usual manner. The prescribed displacements are implemented by replacing the diagonal element of the corresponding row by unity and the rest of the element of the rows and columns by zeros [49].

1. Incremental Method (INC)

Consider the incremental process, we can rewrite the equation of load-displacement relationship,

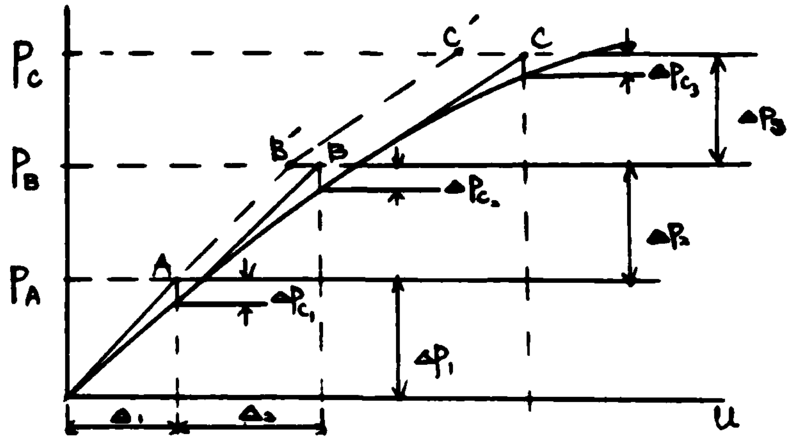
$$[K]_i\ \{\Delta\}_{i+1} = \{\Delta P\}_{i+1} + \{\Delta P_c\}_i \quad (2.29)$$

where, i : the current configuration coordinates.

$\{\Delta P_c\}_i$: the force unbalance

$\{\Delta P\}_{i+1}$: an increment in the external load

The method is described by the Figure 2.6



where, ABC : The algorithm of equation (2.29)

AB' C' : The algorithm of purely incremental process

Figure 2.6. The Algorithm of Incremental process

The approximate values of A, B, C, is computed from $p = f(u)$. To improve these values, take the iteration process at each load level with $\{\Delta P\}_{i+1} = 0$, as is Figure 2.7.

The iteration processes are two ways, one is few load levels with many corrective iterations in each, or another is many load levels with few iterations in each. This is sometimes called "incremental with one-step Newton-Raphson method".

2. Full Newton-Raphson Method (FNR)

Consider the load versus displacement function $P = f(u)$. Take the Taylor series of $P = f(u)$

$$f(u_A + \Delta_1) = f(u_A) + \left(\frac{dP}{du}\right)_A \Delta_1 + \cdots \qquad (2.30)$$

where, $\left(\frac{dP}{du}\right)_A = K_A =$ tangent stiffness at A

$$f(u_A) = P_A$$

$$f(u_A + \Delta_1) = P_B$$

We look for Δ_1 such that $f(u_A + \Delta_1) = P_B$, so

$$K_A (\Delta_1) = P_B - P_B \qquad (2.31)$$

The consequent steps are explained by the Figure 2.7.

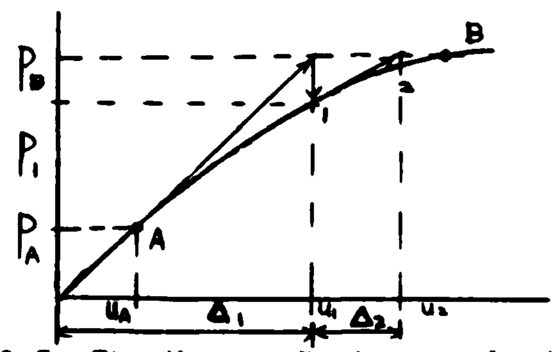


Figure 2.7. The Newton Raphson solution of the equation $p = f(u)$

The above algorithms are below;

. update displacements : $u_1 = u_A + \Delta_1$

. use u_1 to get stiffness k_1 and load P_1

. find next increment Δ_2 from $k_1 (\Delta_2) = P_B - P_1$

Eventually $u_A + \Delta_1 + \Delta_2 + \ldots\ldots = u_B$ to a close approximation.

Then, the load can be increased from P_B to P_C, and the process can be started again. This solution algorithm is

known as the Newton-Raphson Method.

3. Modified Newton-Raphson Method (MNR)

The Newton-Raphson method requires that tangent stiffness matrix [k] be generated and reduced for equation solving in every iteration cycle. This process is not economic. An alternative is modified Newton-Raphson iteration, also called "constant stiffness iteration". Here the same tangent stiffness matrix is used for several iterations cycles. The MNR is described by the Figure 2.8

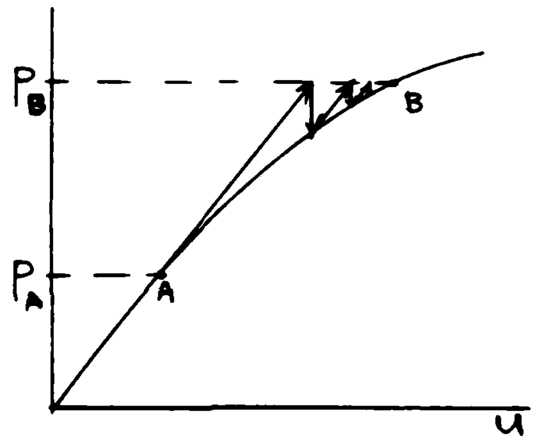


Figure 2.8 The MNR Iteration

As compared with FNR, the MNR requires more iterations, but each is done more quickly because [K] is formed and reduced in only the first iteration.

E. ANALYTIC SOLUTIONS

At first we define the concept of stresses [52] Eulerian stress, σ , is defined as follows:

$$\sigma\ (y,t) = \frac{N(y,t)}{A(y)} \qquad (2.32)$$

and is called True stress. Lagrangian stress, T, is defined as follows:

$$T(x,t) = \frac{N(y,t)}{A_o(x)} \qquad (2.33)$$

and is called Engineering stress.

Kirchloff stress, S, is defined as follows:

$$S(x,t) = \frac{N(y,t)}{\lambda(x,t)\ A_o(x)} \qquad (2.34)$$

The relationship between strain r , and stretch is as follows:

$$r = \tfrac{1}{2}\ (\lambda^2 - 1) \qquad (2.35)$$

$$\lambda = 1 + \frac{du}{dx} \qquad (2.36)$$

where, N : Applied force

A : deformed area

For the special case of small strain, both u, x and change in cross-sectional area are small. In this case, we can write

$$r \approx u_{,x} = \epsilon$$
$$\sigma \approx T \approx S$$

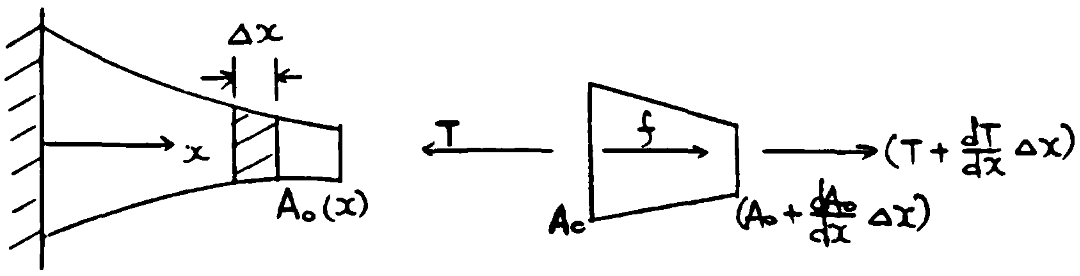
where, ϵ is engineering strain.

Thus in small deformation case, the Lagrangian stress T and engineering strain ϵ can be used to replace the

and engineering strain ϵ can be used to replace the Kirchhoff stress S and Lagrangian strain Γ .

1. Elastic Materials

Consider a one-dimensional rod with varying cross- sectional area, $A_o = A_o(x)$, in equilibrium under distributed external loading.



(a) (b)

where, f : external distributed load per unit length

T : Lagrangian stress within rod

Figure 2.9. One-Dimensional Rod under Axial Loading

For static problems and elastic materials, both f and T are independent of time,

ie. $f = f(x)$ and $T = T(x)$

From figure 2.9(b), the static equilibrium of forces in the x-direction is

$$\Sigma F(x) = -TA_0 + f\Delta x + (T + (dT/dx)\Delta x)(A_0 + (dA_0/dx)\Delta x) = 0 \quad (2.37)$$

Expanding in Taylor series,

$$f + T\frac{dA_o}{dx} + A_o\frac{dT}{dx} + \frac{dT}{dx}\frac{dA_o}{dx}\Delta x = 0 \qquad (2.38)$$

For a continuous rod, in the limit,

$$f + \frac{d}{dx}(TA_o) = 0 \qquad (2.39)$$

Substitution of equations (2.34), (2.35), and (2.36) in the above (2.40)

where, $u(x)$: displacement w.r.t. x

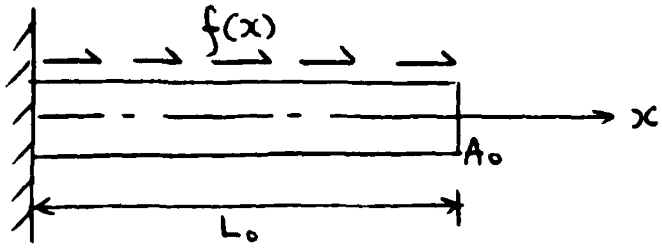
$u_{,x}$: first derivative of $u(x)$

$u_{,xx}$: second derivative of $u(x)$

Note that equation (2.40) gives the values of distributed force f (per unit length) required to produce the displacement field $u(x)$ of a one-dimensional rod once the boundary conditions, the stress-strain law, and varying area function $A_o(x)$ have been specified. Consider the rod with constant area A_o, equation (2.40) becomes

$$f(x) = -A_0\, u_{,xx}\ [(1 + u_{,x})^{**}2\,(dS^*/dr) + S^*] \qquad (2.41)$$

Some special cases of equation (2.41) is considered. One case is the specified displacements.



where, $u(x) = a_0 + a_1 x + a_2 x^2$

$u_{,x} = a_1 + 2 a_2 x$

$u_{,xx} = 2 a_2$

Figure 2.10 Test Model of "Inverse Solution"

Apply the boundary contition $u_{,x}(x) = 0$ at $x = L_0$

$$u(x) = a_1 x - \frac{a_1}{2L_0} x^2 \tag{2.42}$$

If a_1 is specified, we may get the exact solution u(x) and the corresponding required distributed load f(x).

a. Linear displacements field:

$u(x) = a_1 x$

$u_{,x} = a_1$

$u_{,xx} = 0$

From equation (2.42), we may get

$f(x) = 0$

Note that a linear displacement field cannot be produced with distributed load.

b. Quadratic displacement field:

$$u(x) = a_1 x - (a_1/2L_o)x^2$$

$$u_{,x} = a_1 - y(a_1/L_o)x$$

$$u_{,xx} = -(a_1/L_o)$$

- Consider the linear materials, $S^e = E\,r$

We may get the distributed forces from equation (2.41)

$$f(x) = -EA_o u_{,xx} \, [(1 + u_{,x}) + (u_{,x} + \tfrac{1}{2} u^2_{,x})] \qquad (2.43)$$

- - Consider the nonlinear materials,

$S^e = E_1 r + E_2 r^2$ (quadratic form)

From equation (2.41), we may get

$$f(x) = -A_o u_{,xx} \{ (1 + u_{,x}) [E_1 + 2E_2 (u_{,x} + \tfrac{1}{2} u^2_{,x})] + E_1 (u_{,x} + \tfrac{1}{2}u^2_{,x}) + E_2 (u_{,x} + \tfrac{1}{2}u^2_{,x})] \} \qquad (2.44)$$

Another case is the prescribed loads.

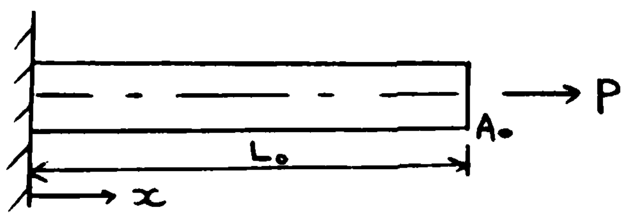


Figure 2.11 Test Model of Rod with Prescribed End Element.

From equation (2.39), since no distributed loads exist,

$$\frac{d}{dx}(TA_o) = 0 \quad \text{with constant area.}$$

We may get

$$S = \text{constant} \qquad (2.45)$$

The applied load is given

$$P = A_o \lambda S \tag{2.46}$$

- Linear material;

$$S^e = Er = E(u_{,x} + \tfrac{1}{2} u^2_{,x})$$

From equation (2.46), we may get

$$P = EA_o \, (u_{,x} + \quad u^2_{,x} + \tfrac{1}{2} u^3_{,x}) \tag{2.47}$$

- Nonlinear material;

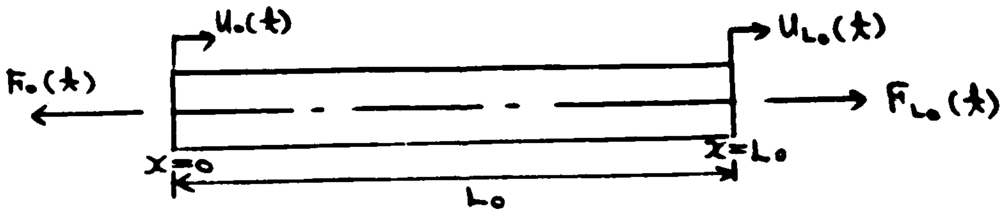
$$S^e = E_1 r + E_2 r^2 \quad \text{(quadratic form)}$$

From equation (2.46) we may get

$$P = A_o \, (1 + u_{,x}) \, [\, E_1 \, (u_{,x} + \tfrac{1}{2} u^2_{,x}) + E_2 \, (u_{,x} + \tfrac{1}{2} u^2_{,x})^2 \,] \tag{2.48}$$

2. <u>Quasilinear Viscoelastic materials</u>

The Viscoelastic materials has the more complicated problems than the elastic materials, since the external loads and displacements are time-dependent.



where, A_o, L_o : Underformed Area and Length

$u_o(t)$: time-varying left-end displacement

$u_{Lo}(t)$: time-varying right-end displacement

$F_o(t)$: time-varying left-end force

$F_{Lo}(t)$: time-varying right-end force

Figure 2.12 One-Dimensional Rod Subjected to Time-varying End Loads

For quasilinear viscoelastic materials,

$$S(t) = S^e(t) - \int_0^t \frac{\partial G(t-\xi)}{\partial \xi} S^e(\xi)\, d\xi \qquad (2.15)$$

The Lagrangian stress is constant throughout the rod since the distributed loads is not applied. From equation (2.39), the stress is time variable only.

$$T(x,t) = g(t) = \lambda(t) \quad S(t) \qquad (2.49)$$

And the internal forces produced with the rod is

$$F(t) = TA_o = A_o g(t) \qquad (2.50)$$

Substituting equation (2.15) in equation (2.49) yields

$$g(t) = \lambda(t) \quad S(t)$$

$$= (1 + u_{,x}) \left[S^e(r) - \int_0^t \frac{\partial G(t-\xi)}{\partial \xi} S^e(r)\, d\xi \right] \qquad (2.51)$$

The right hand side of equation (2.51) is a time function only.

Therefore $g(t)$ must also be a time function.

$$\frac{du}{dx} = u_{,x}(x,t) = h(t) \qquad (2.52)$$

Integrating w.r.t. x at equation (2.52)

$$u = u(x,t) = xh(t) + C(t) \qquad (2.53)$$

where, C : integrating function to be determined by the boundary condition at one end.

Assume that

$u(o,t) = u_o(t) = C(t)$,

therefore

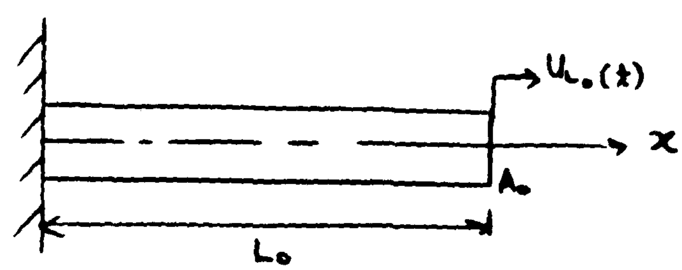
$$u(x,t) = xh(t) + u_o(t), \tag{2.54}$$

$$u_{,x}(x,t) = h(t)$$

Substituting equation (2.54) in equation (2.51) yields

$$\{1 + h(t)\}\left[S^e\{h(t) + \tfrac{1}{2}h^2(t)\} - \int_0^t \frac{\partial G(t-\xi)}{\partial \xi} S^e\{h(\xi) + \tfrac{1}{2}h^2(\xi)\}d\xi\right] \tag{2.55}$$
$$= g(t)$$

Once the G(t) and S*(r) have been specified, the equation (2.55) can be used with the boundary conditions to find the exact solution in some cases. Consider the 3 element solid with linear material property. The prescribed displacements boundary conditions at both ends is described in Figure 2.13.



B.C.'S : $u(o,t) = u_o(t) = 0$

$$u(L_o,t) = u_{L_o}(t) \tag{2.56}$$

Figure 2.13 Test Model of a Rod with Prescribed Time Varying Displacement at the End.

The prescribed displacements are linear,

$$u(x,t) = (x/L_o)\, u_{L_o}(t) \tag{2.57}$$

$$u_{,x}(x,t) = u_{L_o}(t) \,/\, L_o = h(t) \tag{2.58}$$

where, $u_{L_o}(t) = L_o h(t)$

Assume linear material property, $S^e = Er$

And for a 3 element rod, G(t) is defined below

$$G(t) = (1 - \alpha) + e^{-\beta t} \tag{2.59}$$

where, α, β, constants for material properties.

Note that $G(o) = 1$ at $t = 0$, $G(t) = 1 - \alpha$ at $\beta = 0$

No relaxation occurs and the material exhibits elastic response.

Specifying the prescribed displacement function $u_{L_o}(t)$.

a. Ramp Function

At $x = L_o$, B. C.'S : $u(o,t) = 0$

$u(L_o,t) = u_{L_o}(t) = at$ (a: constant)

From equation (2.57) and 2.58), we may get

$$u(x,t) = (a/L_o)\, x\, t \tag{2.60}$$

$$u_{,x}(x,t) = (a/L_o)\, t$$

Substituting equations (2.57) - (2.59) in equation (2.55)

We may get the reaction force, F.

$$F = \frac{EA_o}{L_o}\left[1 + \frac{at}{L_o}\right]\left[\left\{at + \frac{a^2t^2}{2L_o}\right\} - \frac{\alpha a}{\beta}\left\{1 - \left(\frac{a}{\beta L_o}\right)\right\}e^{-\beta t}\right.$$

$$\left. - \frac{\alpha a}{\beta}\left\{(\beta t - 1) + \frac{a}{2\beta L_o}(\beta^2 t^2 - 2\beta t + 2)\right\}\right] \tag{2.61}$$

Equation (2.61) describes the reaction force produced at the fixed end for the prescribed displacement

$u_{L_o}(t) = at$

b. Step Function (Relaxation Test)

At $x = L_o$, B.C.'S : $u(o,t) = 0$

$$u(L_o, t) = u_{L_o} 1(t)$$

where, $1(t)$. unit step function

From equation (2.57) and (2.58), we may get

$$u(x,t) = \frac{u_{L_o}}{L_o} x \, 1(t) \tag{2.62}$$

$$u_{,x} = \frac{u_{L_o}}{L_o} 1(t) \tag{2.63}$$

Substituting equations (2.57) - (2.59) and (2.62) in equation (2.55), we may get the reaction force, F.

$$F(t) = \left(\frac{EA_o}{L_o}\right) u_{L_o} \left(1 + \frac{u_{L_o}}{2L_o}\right) G(t) \left[1 + \left(\frac{u_{L_o}}{L_o}\right) 1(t)\right] \tag{2.64}$$

Consider the Kirchhoff stresses.

From equation (2.34)

$$S = E\left(\frac{u_{L_o}}{L_o}\right)\left(1 + \frac{u_{L_o}}{2L_o}\right) G(t) = E\, r_{L_o}\, G(t) \tag{2.65}$$

where, $r_{L_o} = \frac{u_{L_o}}{L_o}\left(1 + \frac{u_{L_o}}{2L_o}\right)$: Lagrangian strain due to a prescribed step displacement function $u_{L_o} 1(t)$ at the rod end.

III. PROGRAM IMPLEMENTATION

A. OVERVIEW

We are interested in static analysis. For static analysis, load-displacement relationship is as following:

$\{P\} = [k]\{u\}$

where $\{P\}$: external loads vector

$[k]$: constrained stiffness matrix

$\{u\}$: nodal displacements vector

If we know $\{P\}$ or $\{u\}$, then we can analyze the static problems by using elementary elastic theory. In general, the static analysis steps are shown as follows:

Step 1 : Enter geometry of material properties, loads, et al.

Step 2 : Calculate element stiffness matrices.

Step 3 : Determine appropriate part of $[k]$ and $\{P\}$.

Step 4 : If $[k]$ or $\{P\}$ is appropriate, then repeat step 2 through step 3.

Step 5 : If $[k]$ or $\{P\}$ is not appropriate, then apply constraints.

Step 6 : Determine $\{u\}$.

Step 7 : Regenerate elements.

Step 8 : Calculate element stresses.

B. SOLUTION PROCEDURES

In nonlinear structural problems, to find the solutions we use the INC and FNR and MNR. In fact, it is hard to know in advance what increments of loads or iteration should be used to obtain a good approximation to the solution. The interactive method was used in determining the strategy for solution in program application. The strategy subroutines were developed for elastic or visco-elastic time-dependent structural problems. Here the initial state for incremental loading process was selected as a null displacement vector. Next we consider finite element method for elastic (FEMEL) and viscoelastic (FEMEV) programs. IN both programs, equilibrium was checked at each stage for solutions obtained from other INC or FNR and MNR.

1. Algorithm Implementation

Consider the algorithm of Newton-Raphson Method based on total Lagrangian Method. Full Newton-Raphson Algorithm is as follows:

Step 1. Establish local coordinates X-Y for the element at hand.

Step 2. Generate the element stiffness matrix [k] in local coordinates so that it operates on the local D.O.F.

Step 3. Transform [k] to global coordinates so that it operates on global D.O.F.

Step 4. Repeat step 1 through 3 until all elements have been

treated.

Step 5. **Compute displacements.**

Step 6. **Compute internal forces,** [k]{u}.

Step 7. Solve [k] $\{\Delta u\} = \{\Delta P\}$

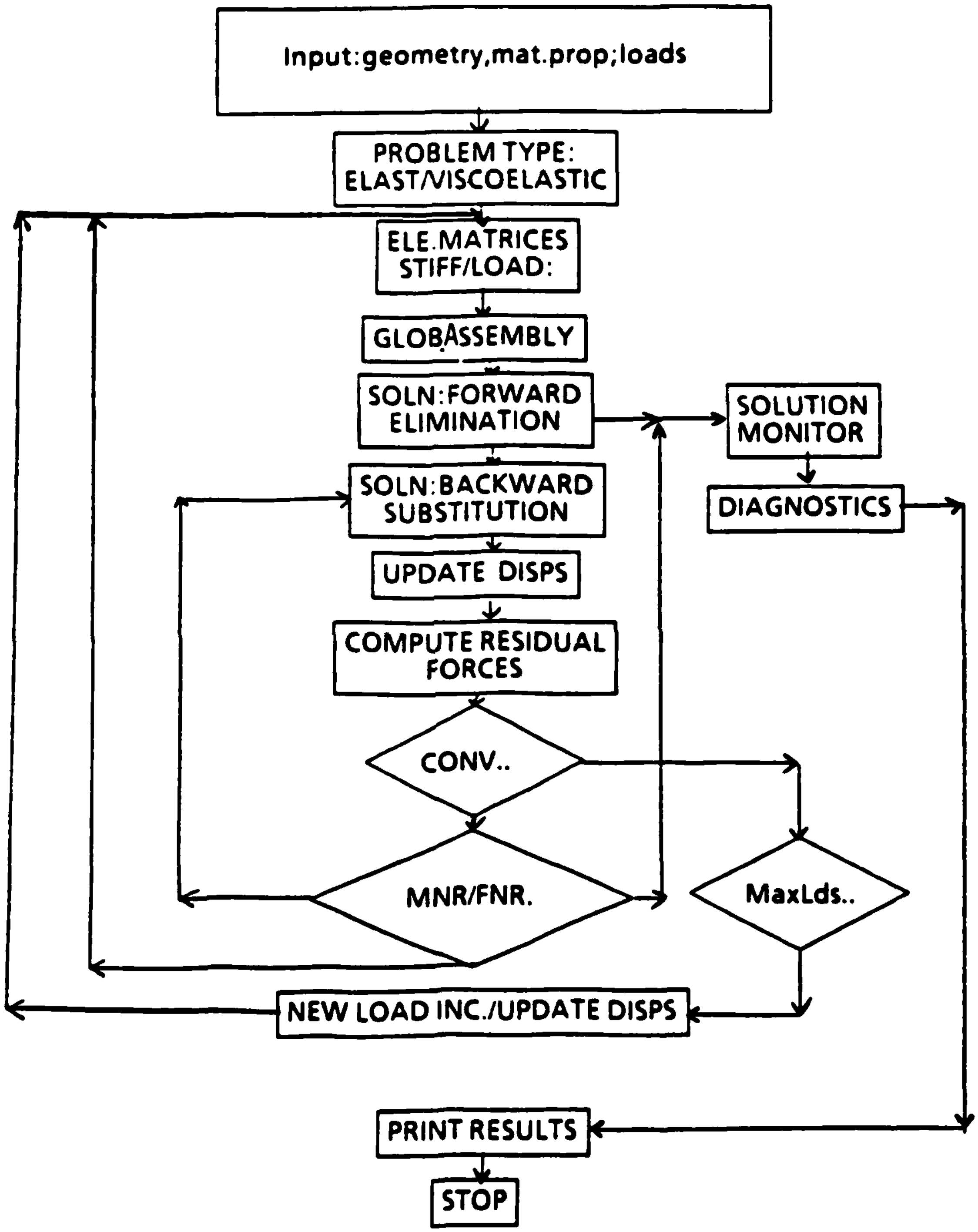
where, $u_{N+1} = u_N + \Delta u_N$

Step 8. Test for convergence.

If not satisfied, return to step 1.

Where, convergence = $((\Delta R^T \Delta R) / (\Delta P^T \Delta P))**0.5 <$ 0.001

2. Flow Chart of the Finite Element Program



Flowchart of the Finite element Program

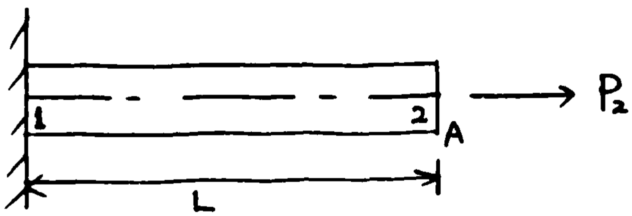
IV. NUMERICAL EXAMPLES

We consider two numerical examples for elastic and viscoelastic materials.

A. ELASTIC MATERIAL

1. Prescribed concentrated forces.

Specify the rod element.



P_2 = 22,5000 lbs

A = 0.25 [in^2]

L = 10 [in]

E = 10^6 [lbs/in^2]

Consider the stiffness matrix of rod element, [k]. For the rod element,

$$[K] = \frac{EA}{L}\begin{bmatrix} K_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \tag{4.1}$$

Consider the boundary conditions,

$u_1 = 0$

$P_1 = 0$

$P_2 = 22,5000$ at $x = L$

From loads - displacements relationship,

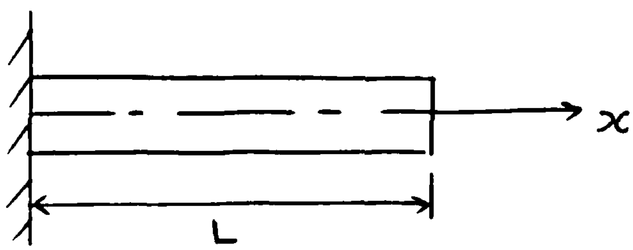
$$\frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} = \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix}$$

Solve for the displacements, u_2,

Hence, we may get

$$U_2 = \frac{LP_2}{EA} = 9.0 \text{ (in)}$$

2. <u>Prescribed displacements</u>



Consider the same rod element before, and specified the quadratic displacements, u(x),

$$u(x) = 0.02\,x - 0.001\,x^2$$

The first derivative is

$$\frac{du}{dx} = 0.02 - 0.002\,x$$

The second derivative is

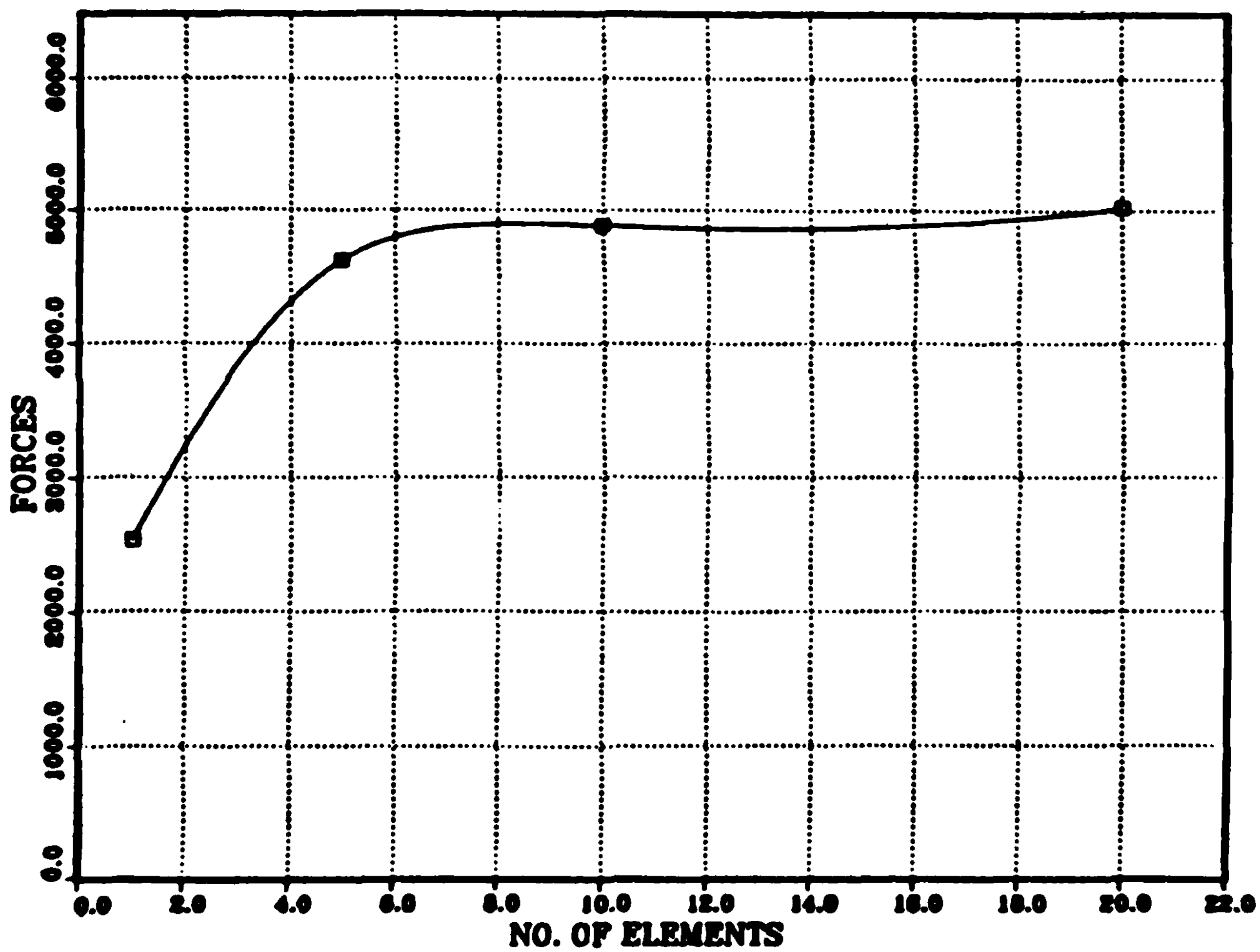
$$\frac{d^2u}{dx^2} = 0.002$$

From equation (2.41) with the constant area, we may getthe distributed loads for any x.

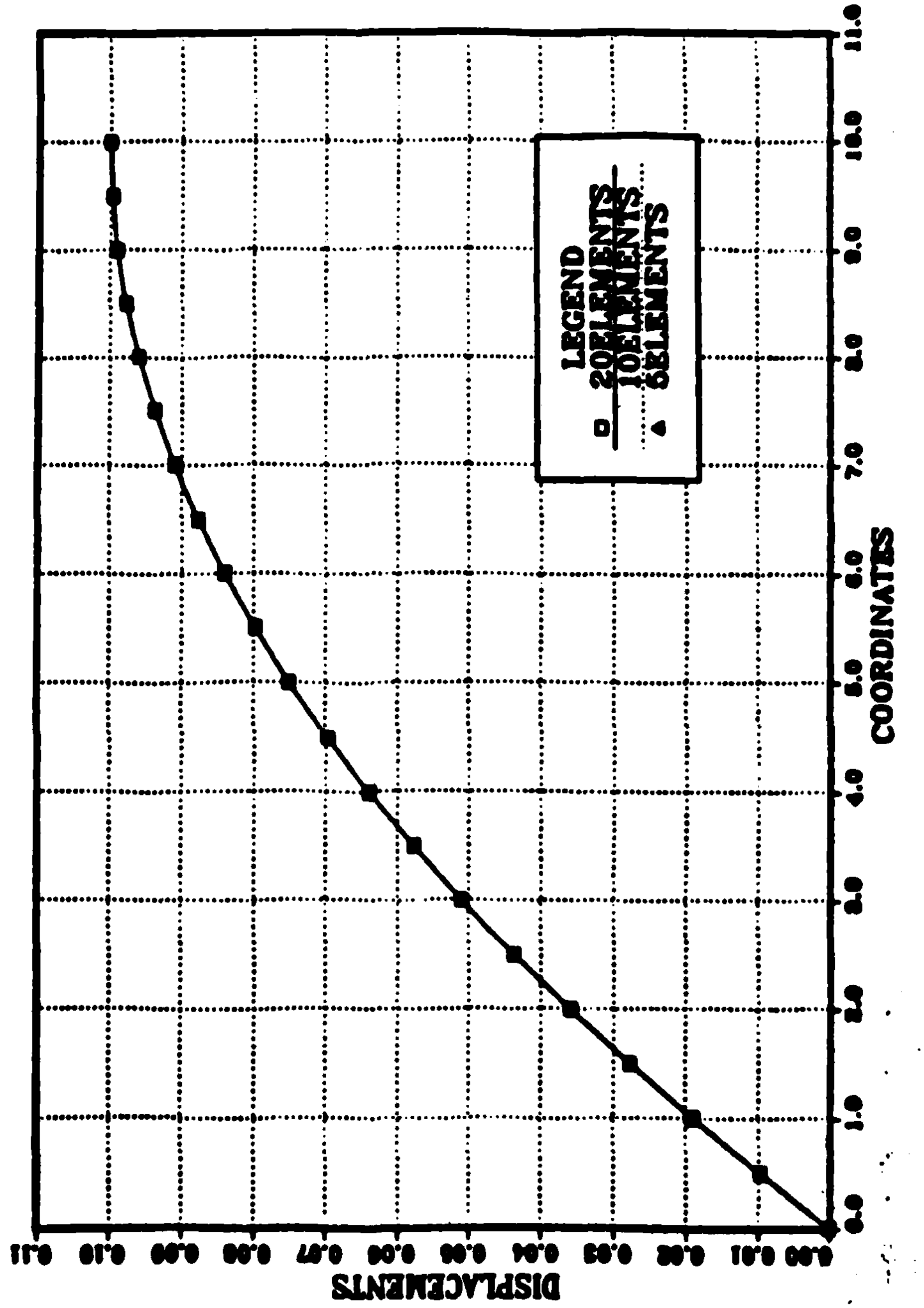
$$f(x) = -A_o \frac{d^2u}{dx^2}\left[\left(1 + \frac{du}{dx}\right)^2 \frac{ds^e}{dr} + S^e\right]$$

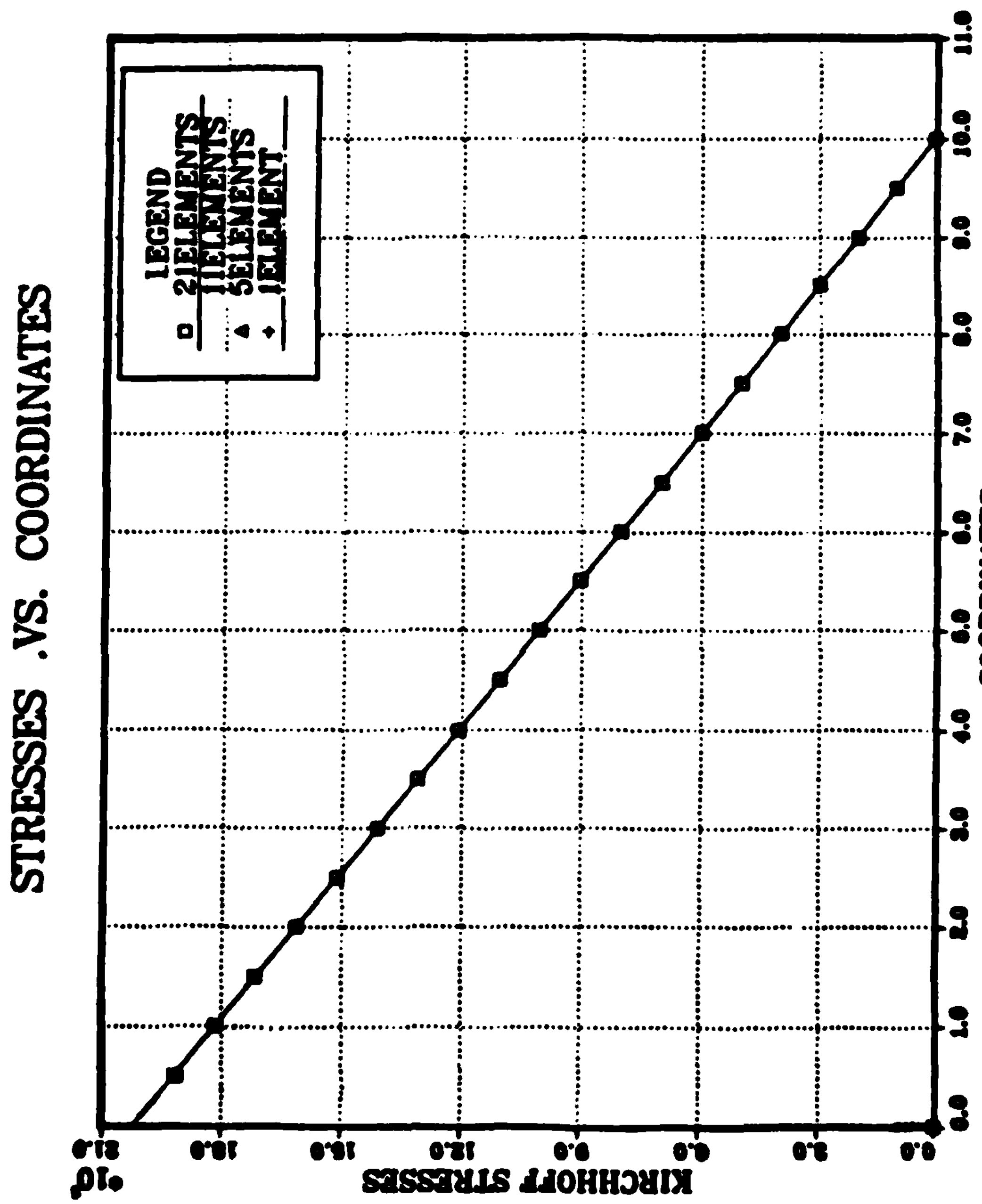
$$= 3.0 * 10^{-3} x^2 - 3.06 x + 530 \qquad (4.2)$$

FORCE .VS. NO. OF ELEMENTS



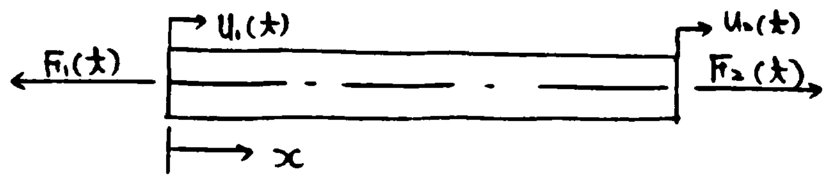
DISPLACEMENTS .VS. COORDINATES




STRESSES .VS. COORDINATES
LEGEND
21ELEMENTS
11ELEMENTS
5ELEMENTS
1ELEMENT
KIRCHHOFF STRESSES
*10^1
0.0
3.0
6.0
9.0
12.0
15.0
18.0
21.0
COORDINATES
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0
11.0

B. VISCOELASTIC MATERIAL

Consider the rod element below,



Boundary conditions; $u(0,t) = u_o(t) = 0$

$$u(L_o,t) = u_{L_o}(t) = u_{L_o}\, 1(t)$$

Lagrangian Stress; $T(x,t) = g(t) = A(t)\, S(t)$

Internal Force; $F(t) = T\, A_o\, g(t)$

Kirchhoff stress; $$S = \frac{P}{\lambda(x, t)\, A_o(x)} \tag{4.3}$$

For 3 element solids, the relaxation function is

$$G(t) = (1 - \alpha) + \alpha\, e^{-\beta t} \tag{4.4}$$

where, α and β are constants of the material properties.

Assume unit step function for prescribed displacements.

At that time, the boundary condition is

$u(o, t) = 0$

$u(L_o, t) = u_{L_o}(t) = u_{L_o}\, 1(t)$

where, $1(t)$: unit step function

We may get the displacements function with unit step function.

$$u(x, t) = \frac{x}{L_o}\, u_{L_o}\, 1(t) \tag{4.5}$$

$$\frac{du}{dx} = \frac{u_{Lo}}{L_o} 1(t) \tag{4.6}$$

And the Kirchhoff stress is specified

$$S = E\, r \tag{4.7}$$

Substituting equations (4.4) - (4.7) in equation (4.3),

$$F(t) = \frac{EA_o}{L_o} U_{Lo}\ 1 + \frac{u_{Lo}}{2L_o}\ G(t)\ \ 1 + \frac{u_{Lo}}{L_o}\ 1(t) \tag{4.8}$$

Hence, we may get the Kirchhoff stress, S,

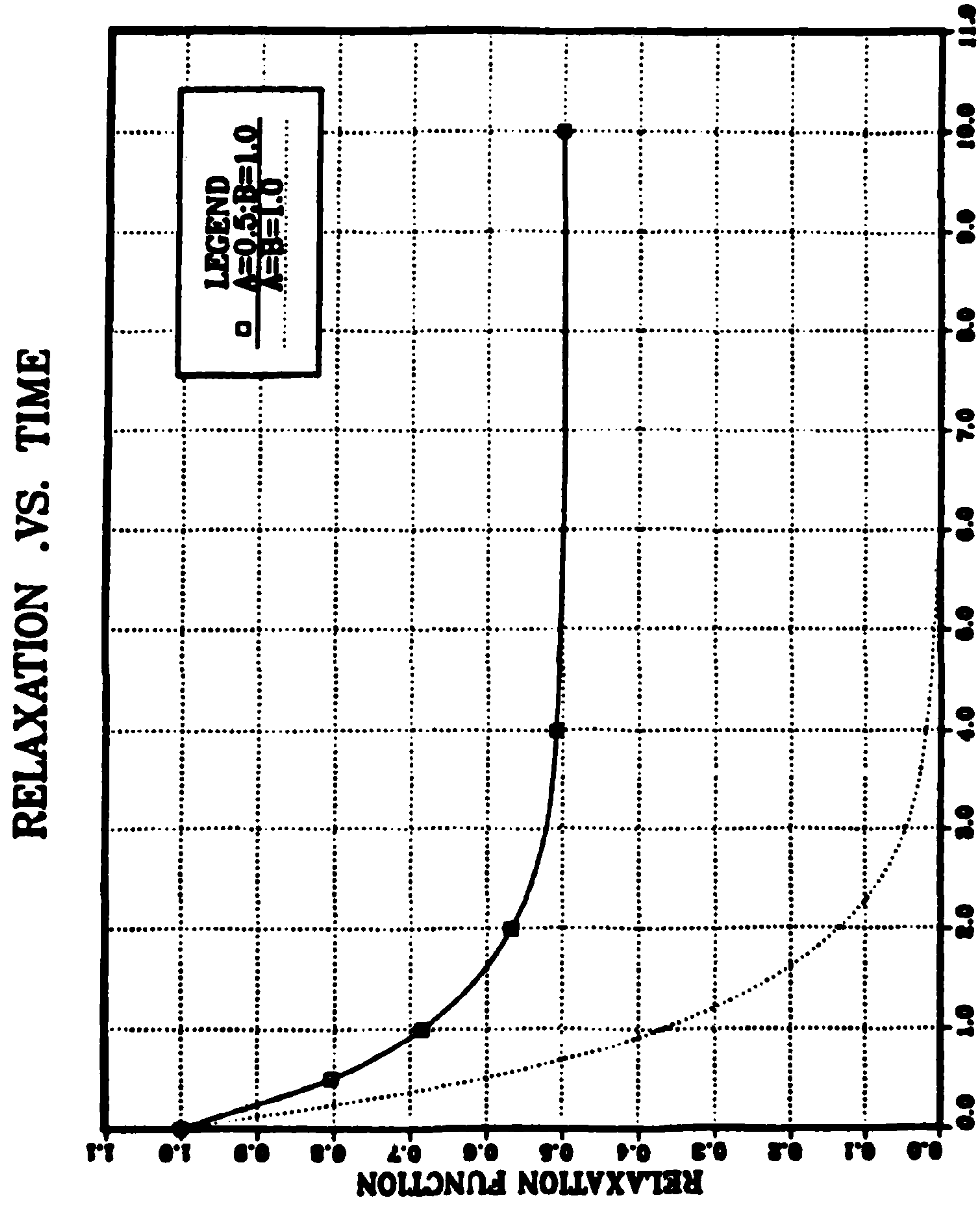
$$S = r_{L_o}\ G(t) \tag{4.9}$$

where, $r_{Lo} = \frac{u_{Lo}}{L_o}\ 1 + \frac{u_{Lo}}{2L_o}$: Lagrangian strain

due to the prescribed step displacement function

u 1(t) at the rod end.

Equation (4.9) describes the relaxation cases. From equation (4.8), we may get the internal forces.


RELAXATION .VS. TIME
LEGEND
A=0.5;B=1.0
A=B=1.0
RELAXATION FUNCTION
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
TIME
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0
11.0

V. CONCLUSION

This study is directed towards understanding the nonlinear behavior of structures.

The formulation is based on the principle of virtual work. We consider the one dimensional nonlinear continuum-based rod element that includes large displacements and viscoelastic effects. To get the solution we use the incremental process with full Newton-Raphson and modified Newton-Raphson Method. Numerical solutions agree well with exact solutions.

Further studies may be extended to two dimensional structures, such as laminated composite plates. Consideration of the viscoelastic effects of composite panels at high temperatures is another possible area of study.

An experimental study to verify the results of the present study is also recommended.

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